SYNOPTIC: Eddy Currents in an Infinitely Finely Segmented Hall Generator,

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Current Distribution; Magnetohydrodynamic Hall Generator

Theme

This paper describes a theoretical study of the current distribution over the cross section of a combustion driven Hall generator. The gas is assumed to be in chemical equilibrium. Under the assumption of weak interaction, the fluid mechanic problem is decoupled from the electromagnetic problem. Comparison with some crude experiments showed favorable agreement.

Content

The cross-sectional current distribution of a Hall generator is studied. The axial variation of all quantities are neglected in comparison with the transverse variations. This assumption implies that the segmentation is infinite so that there is no current concentration on the electrode. The regimes in which open cycle MHD generators operate are confined to small magnetic Reynolds number and weak interaction. The former condition allows the neglecting of the induced magnetic field, while the later condition makes it possible to decouple the fluid dynamic problem from the electromagnetic problem.

The governing equations are the generalized Ohm's law and the Maxwell's equations

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - (\Omega/|\mathbf{B}|)(\mathbf{j} \times \mathbf{B}) \tag{1}$$

$$\nabla \cdot \mathbf{j} = 0 \tag{2}$$

$$\nabla \times \mathbf{E} = 0 \tag{3}$$

where $\mathbf{j}$ is the current density vector, $\mathbf{E}$ the electric field vector, $\mathbf{V}$ the plasma velocity, $\mathbf{B} = B_0\mathbf{k}$ is the applied magnetic field which is a constant, σ the electrical conductivity, and Ω the Hall parameter.

From Eq. (3) and the assumption of no axial variation, e.g., $\partial/\partial x = 0$, it follows $\partial E_x/\partial x = \partial E_x/\partial y = \partial E_x/\partial z = 0$ thus $E_x =$ const. A partial electric potential φ is introduced as

$$E_y = -\partial\varphi/\partial y \text{ and } E_z = -\partial\varphi/\partial z \tag{4}$$

Eliminating $\mathbf{j}$ and $\mathbf{E}$ from Eq. (1) with the aid of Eqs. (2) and (4), we obtain a second-order partial differential equation for φ

$$\frac{\partial^2\varphi}{\partial y^2} + (1 + \Omega^2)\frac{\partial^2\varphi}{\partial z^2} + \left[\frac{\partial \ln\sigma}{\partial y} - \frac{\partial \ln(1 + \Omega^2)}{\partial y}\right]\frac{\partial\varphi}{\partial y} + (1 + \Omega^2)\frac{\partial \ln\sigma}{\partial z}\frac{\partial\varphi}{\partial z} + [\Omega(E_x - vB_0) - uB_0]\frac{\partial \ln\sigma}{\partial y} - B_0\left(\frac{\partial u}{\partial y} + \Omega\frac{\partial v}{\partial y}\right) + \left\{(E_x - vB_0) + \frac{2\Omega}{1 + \Omega^2}[uB_0 - \Omega(E_x - vB_0)]\right\}\frac{\partial\Omega}{\partial y} = 0 \tag{5}$$

The boundary conditions are that φ is constant on the walls (cathode, anode, and two side walls).

For a combustion gas plasma, the electrons are in thermal equilibrium with the gas which is assumed to be not influenced by the presence of the currents and fields. Therefore, Eq. (5) can be solved numerically with known distributions of the gas properties. Both fully developed laminar flow and turbulent profiles are used for the computation. It was found that the wall temperature has very strong influence on the laminar flow condition, with the cold wall current distribution being much more nonuniform. Under turbulent boundary-layer flow conditions, however, cold and hot wall current distributions exhibit relatively minor differences, with both cases approaching the constant property limit. Increasing of the magnetic field strength and the Hall parameter tends to increase the nonuniformity of the current distribution. The power output of the Hall generator is shown to depend strongly on the wall temperature for laminar flow, but this dependence is greatly reduced when the turbulent boundary-layer velocity profile is assumed.